StructInf: Mining Structural Influence from Social Streams

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Abstract

Social influence is a fundamental issue in social network analysis and has attracted tremendous attention with the rapid growth of online social networks. However, existing research mainly focuses on studying peer influence. This paper introduces a novel notion of *structural influence* and studies how to efficiently discover structural influence patterns from social streams. We present three sampling algorithms with theoretical unbiased guarantee to speed up the discovery process. Experiments on a big microblogging dataset show that the proposed sampling algorithms can achieve a $10\times$ speedup compared to the exact influence pattern mining algorithm, with an average error rate of only 1.0%. The extracted structural influence patterns have many applications. We apply them to predict retweet behavior, with performance being significantly improved.

Introduction

Social influence occurs when one's behaviors or opinions are affected by others. It forms a fundamental mechanism governing the dynamics of social networks, and recently, has attracted tremendous attention with the availability of large online social behavior data. For example, a field experiment conducted on Facebook shows strong evidence of social influence on political mobilization (Bond et al. 2012). The theory of three degrees of influence (Fowler, Christakis, and others 2008) claims that our behavior can influence people we have never met. However, the underlying mechanism is still unclear and a thorough investigation is thus needed.

We show an example of retweet behavior influence in three different structures on a dataset of Sina Weibo[1] in Figure 1. The red node represents a neighboring user (followee in Weibo) who retweets a message before time t; the white node denotes the target user to be studied. The general question is how likely it is that the target user will retweet this message at time t' ($0 < t' - t \leq \tau$, τ is a short time interval), conditioned on different influence structures. We can see several interesting patterns. First, the conditional probability in Figure 1(b) increases to 150% higher than that of Figure 1(a), suggesting more active neighbors can improve the retweet likelihood. On the other hand, the probability

[1]The most popular Chinese microblogging site (weibo.com).

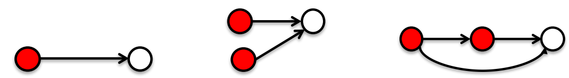


Figure 1: Probability that the target user (white node) retweets a message on Weibo, conditioned on their neighbors (red nodes) having already retweeted the message.

in Figure 1(c) increases to 300% higher than that of Figure 1(b). The difference between them is the relationship between the neighbors. Please note that the target user may be unaware of such a relationship. How does the network structure formed by friends matter in influencing users' behavior changes? What are the most significant influence structures hidden in the huge volume of streaming behavior data? Although social influence has been extensively studied before (Anagnostopoulos, Kumar, and Mahdian 2008; Kempe, Kleinberg, and Tardos 2003; Tang et al. 2009), most of the existing work focused on peer influence, and ignored the effect of such influence structures. Ugander et al. (2012) presented the idea of structural diversity and showed that diverse structures of neighboring users can influence user's behaviors. However, they only focused on investigating qualitative effects of social influence, but ignored quantitative estimation.

In this work, we formalize the problem of *mining structural influence* from the social stream. Specifically, given large user behavior logs and the network structure among users, how can we discover influence structures with high confidence? The discovered influence structures can be applied in many different applications, e.g., to be used as features in a paper's citation network to predict whether the paper will be extensively cited (Shi, Leskovec, and McFarland 2010), or to predict one's "following" behaviors in a user's "following" network (Zhang et al. 2015).

We address the above issue and make the following contributions: (1) we formally define a novel notion of **structural influence**; (2) to handle large streaming behavior data, we propose an exact and several **sampling algorithms** to quickly estimate structural influence; we provide theoretical unbiased guarantees for the sampling algorithms; (3) our empirical study on a large Weibo dataset, show that the proposed sampling algorithms achieve a $10\times$ speedup and result in an average error rate of only 1.0%, compared to our exact counting algorithm; (4) finally, we demonstrate the effect of structural influence on **retweet behavior prediction**.

Problem Formulation

Let $G = (V, E)$ denote a social network, where V is a set of users and $E \subset V \times V$ is a set of relationships. We use $v_i \in V$ to represent a user and $e_{ij} \in E$ to represent a relationship between v_i and v_j. A relationship can be either directed or undirected. With a directed relationship e_{ij}, we only consider one-way influence from v_i to v_j. An undirected relationship can be divided into two directed relationships, e_{ij} and e_{ji}, and the influence is also two-way. We use $\mathcal{N}(v_i)$ to indicate the neighbors of user v_i. Another input of our problem is a stream of user action logs.

Definition 1 ***Action Logs.*** *Let L denote a stream of action logs, where each log entry $l \in L$ is a triple (v, a, t), representing user $v \in V$ performed action $a \in A$ at time t. Here A is a set of action types.*

For example, in a microblogging site, a retweet behavior is an action, and each tweet can be considered as an action type. We build an *action diffusion graph* from G and L.

Definition 2 ***Action Diffusion Graph.*** *An action diffusion graph, $G^p = (L, E^p)$, is a directed graph, where each node is a log entry in L, and each edge $e^p_{ij} \in E^p$ between two log entries (v_i, a, t_i) and (v_j, a, t_j) satisfies 1) $(v_i, v_j) \in E$ and 2) $0 < t_j - t_i \leq \tau$, where τ is a short time interval.*

We use $\mathcal{N}^p(l)$ to indicate the neighbors in G^p that point to l. In an action diffusion graph, an edge e^p_{ij} represents that when user v_i performs action a at time t_i, v_i has a potential influence on her neighbor v_j to perform a at t_j, a short time interval τ after t_i. We define (v_i, a, t_i) as influencing action and (v_j, a, t_j) as target action. If later (v_j, a, t_j) actually happens in L, we call it as *active target action*; otherwise *inactive target action*. Different from conventional research that mainly decomposes the influence from neighbors into peer influences, we propose a new notion as:

Definition 3 ***Structural Influence.*** *When the target user v_j performs action a at t_j and her γ-degree ($\gamma \geq 1$) friends who already performed the action before t_j ($0 < t_j - t_i \leq \tau$, t_i is the latest time when a friend performed the action), we define structural influence as the combination effect of peer influences exerted by those γ-degree active friends on the target user, when the friends and target user form an influence structure. Table 1 lists all the structures when considering 1, 2, and 3 active γ-degree friends. We also name influence structure as structural influence pattern and use C_k to represent the k-th pattern. An instance of C_k for any action is named a pattern instance, and is denoted by c^k_i.*

Structural influence can be formulated as a conditional probability, $\text{IP}_k = \frac{x_k}{x_k + y_k}$, where x_k represents the frequency of the instances with pattern C_k and target action l_t being active in L, while y_k is that of C_k's instances with l_t being inactive in L. Given the above definition, the key question we want to answer is to efficiently discover the structural influence patterns with high influence probability. Note that a node may be influenced by multiple pattern instances, and this combination effect can be modeled by a partial credit model or a cascade model (Goyal, Bonchi, and Lakshmanan 2010). This paper simply follows the assumption of Bernoulli distribution (Goyal, Bonchi, and Lakshmanan 2010) to estimate structural influences from different pattern instances independently. Other assumptions will be studied in the future. Note also that when enumerating an instance, we assign the maximal matched pattern for it. For example, if an instance can be matched to C_4 in Table 1, the sub patterns of C_4, such as C_2 and C_3, will not be matched.

Later, we aim at quickly estimating IP_k for each influence pattern. One potential application is to incorporate the patterns of high probabilities as features to predict user behaviors, and the details will be given in the experimental section.

StructInf: Structural Influence Estimation

In this section, we begin by introducing an exact algorithm to estimate the structural influence from social streams, based on which we propose three fast sampling strategies.

StructInf-Basic

For each pattern C_k, the goal is to estimate x_k and y_k, and based on which to calculate structural influence of C_k. Assume the social network is static, while the action logs are very large and arrive in real time. Our approach loads the static network into memory at the beginning and update x_k and y_k whenever an action log arrives[2]. The key idea of the approach is to 1) identify active and inactive target actions; and 2) enumerate the structural influence patterns from the target actions backwards along the diffusion edges.

Identifying Target Actions. We propose Algorithm 1 to estimate x_k and y_k. We maintain an action diffusion graph G^p, a queue Q and a hashtable H, to record the diffusion edges and action logs within recent time interval τ. For each newly arrived action, (v_i, a_k, t_i), we first add it into Q and H (Line 4), and then update $\vec{x}$ (Lines 5-8) and $\vec{y}$ (Lines 9-19).

To estimate $\vec{x}$, we need to figure out active target actions. Obviously, each newly arrived action is an active target action. Thus when each action $l_t = (v_i, a_k, t_i)$ arrives, we first find those neighbors of v_i who are active in $[t_i - \tau, t_i]$, and create the new diffusion edges from the actions of those active neighbors to l_t in G^p (Lines 5-7), and then enumerate the structural influence patterns starting from l_t (Line 8).

To estimate $\vec{y}$, we need to figure out inactive target actions. To achieve this, we enumerate the influence patterns when an action is outdated rather than newly arrived, because for any action (v, a, t), we can only know whether user v's neighbors are active or not within $[t, t + \tau]$ until time $t + \tau$, which is exactly the time that (v, a, t) is popped up from Q. In particular, whenever a new action arrives, we remove the outdated actions from Q and H (Lines 9-10). For each removed action $l_r = (v_j, a_h, t_j)$, we identify the neighbors of v_j that are inactive in $[t_j, t_j + \tau]$ (Lines 11-12), and assign a virtual time $t_j + \tau$ to each inactive neighbor(Line

[2]In our implementation, storing a static network with millions of nodes and edges costs about 5G memory.

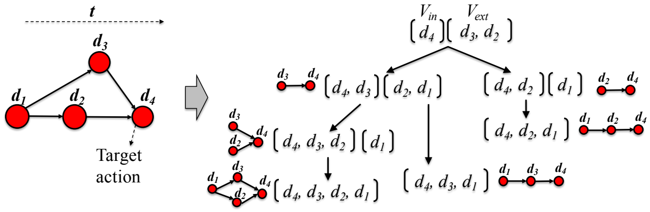


Figure 2: Illustration of pattern enumeration.

13), to create an inactive target action l_t. After that, we add a virtual diffusion edge from l_r to l_t into G^p (Line 14). Finally, we complete the diffusion edges from other active actions to l_t (Lines 15-17). After enumerating from l_t (Line 18), we remove the created virtual diffusion edges (Line 19).

Enumerating Influence Patterns. Algorithm 2 presents how to enumerate all possible influence patterns from a given action log. The basic idea is: we start by adding an active or inactive target action into V_{in}, and their neighboring actions into V_{ext}. Then we extend V_{in} by selecting an arbitrary action l' from V_{ext}, and update V_{ext} by adding all the neighbors of l' that are not included in V_{in}.

One thing worth noting is the necessary of avoiding duplicate enumeration. To achieve this, we assign each action an incremental (unique) label when it arrives (Line 3 in Algorithm 1), and make sure that, anytime, the labels of the actions in V_{ext} are smaller than those in V_{in} (Line 7 in Algorithm 2). Figure 2 illustrates the enumeration process under this strategy. The left part demonstrates an action diffusion graph with all the nodes denoted by incremental labels over time. According to Algorithm 2, we first add the target action d_4 into V_{in}, and d_4's neighbors, namely d_3 and d_2, into V_{ext}, and then extend V_{in} by selecting the actions from V_{ext} one by one. We can see that when selecting d_2 into V_{in}, d_3 is removed from V_{ext} because the label of d_3 is larger than that of d_2 in V_{in}. A similar idea of enumerating subgraphs was used in (Wernicke 2006). They studied static networks, while we are dealing with the streaming behavior data, and thus the derived action diffusion graph evolves over time.

When invoking Algorithm 2, we first induce a subgraph by including all the edges between nodes in V_{in}, and then determine which pattern the induced graph belongs to (Line 2). This is essentially a problem of graph isomorphism. When the pattern is small, we can use the number of nodes/edges and degree sequences, to uniquely identify a pattern, and leverage approximate solutions when the pattern gets larger (Leskovec, Singh, and Kleinberg 2006). The enumeration stops when the considered maximal number of nodes, i.e., N, is reached (Lines 3-4).

Discussions. In summary, the proposed StructInf-Basic algorithm is a streaming approach that only needs one-time scan of the streaming action logs and can carefully avoid duplicate enumeration by a dynamic labeling mechanism. Some other methods proposed by (Kashtan et al. 2004) and (Yan and Han 2002) extend neighboring edges of the selected edges, rather that extending nodes. However, when extending edges, the speeding up is not easy as that of extending nodes (Wernicke 2006). Regarding the time complexity, in Algorithm 1, the time complexity for enumerating active target actions is $O(|L|d_{max})$, and is $O(|L|d^2_{max})$ for inactive target actions, where d_{max} is the maximal degree of G. The complexity of Algorithm 2 is $O((d^p_{max})^N)$, where d^p_{max} is the maximal degree of G^p and N is the maximal number of nodes in all the considered influence patterns. In summary, the total time complexity is $O(|L|d^2_{max}(d^p_{max})^N)$.

Algorithm 1: StructInf-Basic

Input: A network $G = (V, E)$, a stream of action logs L.
Output: $\vec{x}, \vec{y}$.
1 Initialize action diffusion graph G^p, queue Q, hashtable H
2 **foreach** $l_t = (v_i, a_k, t_i) \in L$ **do**
3 Assign an incremental label $d(l_t)$ to l_t;
4 Add l_t into Q and H;
5 **foreach** $v_j \in \mathcal{N}(v_i)$ **do**
6 **if** *H contains* $l_s = (v_j, a_k)$ **then**
7 Add the edge $l_s \rightarrow l_t$ into G^p;
8 $\vec{x}$ += EnumInfPattern(G^p, $\{l_t\}$, $\mathcal{N}^p(l_t)$, []);
9 **while** $l_r = (v_j, a_h, t_j) = Q.head()$ *and* $t_i - t_j > \tau$ **do**
10 Remove l_r from Q and H ;
11 **foreach** $v_m \in \mathcal{N}(v_j)$ **do**
12 **if** *H does not contain* $l_t = (v_m, a_h)$ **then**
13 Set $t_m = t_j + \tau$;
14 Add the edge $l_r \rightarrow l_t$ into G_p;
15 **foreach** $v_n \in \mathcal{N}(v_m)$ **do**
16 **if** *H contains* $l_s = (v_n, a_h)$ **then**
17 Add the edge $l_s \rightarrow l_t$ into G_p;
18 $\vec{y}$ += EnumInfPattern(G^p, $\{l_t\}$, $\mathcal{N}^p(l_t)$, []);
19 Remove edges associated with l_t from Q^p;

Algorithm 2: EnumInfPattern

Input: Action diffusion graph G^p, selected actions V_{in}, extended actions V_{ext}, influence pattern statistics $\vec{n}$.
Output: $\vec{n}$.
1 **if** $|V_{in}| > 1$ **then**
2 c = InducePattern(V_{in}); pid = GetPatternID(c); $n[pid]$++;
3 **if** $|V_{in}| = N$ **then**
4 return $\vec{n}$;
5 **foreach** $l \in V_{ext}$ **do**
6 $V_{in} \leftarrow V_{in} \cup \{l\}$;
7 $V'_{ext} \leftarrow \{l' \in \mathcal{N}^p(l) \cap \overline{V}_{in} \cup V_{ext} : d(l') < d(l)\}$;
8 EnumInfPattern(G^p, V_{in}, V'_{ext}, $\vec{n}$);

StructInf-S: Fast Sampling Algorithms

StructInf-S1. The time complexity of StructInf-Basic is high because it enumerates all possible influence patterns for each target action. To speed up the algorithm, we propose several sampling strategies. The basic requirement is to guarantee unbiased estimation of $\hat{x}_k$ and $\hat{y}_k$. To achieve this, outside Line 8 and 18 in Algorithm 1, we add judgment statements to determine whether target action l_t will be enumerated by a probability p respectively. In the same way, we judge whether a neighboring action l will be enumerated by p outside Lines 6-8 in Algorithm 2. We obtain the lemma:

Lemma 1 *Given an influence pattern C_k, any of its instances, c_i^k, is selected uniformly according to probability p^{n_k}, where n_k is the number of nodes in C_k.*

Proposition 1 *Let x_k be the exact number by StructInf-Basic, and $\hat{x}_k$ be the approximate number by StructInf-S1, then $\widetilde{x}_k = \hat{x}_k / p^{n_k}$ is an unbiased estimator for x_k.*

Proof 1 *The expectation of $\widetilde{x}_k$ can be written as*

$$E(\widetilde{x}_k) = \sum_{n=1}^{S} p(s_n)(\hat{x}_k/p^{n_k})_{s_n}$$

where s_n represents the n^{th} sample of pattern instances and there are S possible samples in the whole pattern space. Notation $(\hat{x}_k/p^{n_k})_{s_n}$ represents the value of $\hat{x}_k/p^{n_k}$ that is estimated in the sample s_n. We can factorize $\hat{x}_k/p^{n_k}$ common to the i^{th} pattern c_i^k and sum over the population:

$$E(\widetilde{x}_k) = \sum_{i=1}^{x_k} \frac{1}{p^{n_k}} \sum_{c_i^k \in s_n} p(s_n)$$

where $\sum_{c_i^k \in s_n} p(s_n)$ indicates the summation over the probabilities of all samples containing the instance c_i^k, which is actually the prior probability that c_i^k is selected in a sample. In StructInf-S1, the probability is p^{n_k}, thus we have

$$E(\widetilde{x}_k) = \sum_{i=1}^{x_k} \frac{1}{p^{n_k}} \times p^{n_k} = x_k$$

According to the sampling theory (Horvitz and Thompson 1952), the unbiased estimation of the variance of $\widetilde{x}_k$ is:

$$\widetilde{V}(\widetilde{x}_k) = \sum_{i=1}^{\hat{x}_k} \frac{1 - p(c_i^k)}{p^2(c_i^k)} + \sum_{i \neq j}^{\hat{x}_k} \frac{p(c_i^k c_j^k) - p(c_i^k)p(c_j^k)}{p(c_i^k c_j^k)p(c_i^k)p(c_j^k)} \quad (1)$$

The above variance is determined by both the approximate estimation, $\hat{x}_k$, and the sampling probability for a pattern, $p(c_i^k)$. According to the central limit theorem, the sum of a random sample of a large enough size from an arbitrary distribution follows approximately a normal distribution, i.e., $\widetilde{x}_k \sim N(x_k, \widetilde{V}(\widetilde{x}_k))$. Thus, the probability of the sampling error with confidence $1-\alpha$ is:

$$p\left[\widetilde{x}_k - z_{\alpha/2}\sqrt{\widetilde{V}(\widetilde{x}_k)} \leq x_k \leq \widetilde{x}_k + z_{\alpha/2}\sqrt{\widetilde{V}(\widetilde{x}_k)}\right] = 1-\alpha$$

where $z_{\alpha/2}$ represents the number of standard deviations, i.e., $\sqrt{\widetilde{V}(\widetilde{x}_k)}$, by which an observation $\widetilde{x}_k$ differs from the mean x_k, when the confidence is $1-\alpha$.

StructInf-S2. Another idea is to first sample diffusion edges uniformly to form a sampled action diffusion graph, and then enumerate the influence patterns based on the sampled graph. Specifically, in Algorithm 1, outside Lines 6-7, we add a judgment statement to determine whether $l_s \rightarrow l_t$ will be added into G^p by a probability q. Outside Lines 12-19 and 16-17, we also determine whether $l_r \rightarrow l_t$ or $l_s \rightarrow l_t$ will be added into G^p by q. We obtain the following lemma:

Lemma 2 *Given an influence pattern C_k, any of its instances, c_i^k, is selected uniformly according to probability q^{m_k}, where m_k is the number of edges in C_k.*

Proposition 2 *Let x_k be the exact number and $\hat{x}_k$ be the approximate number obtained by StructInf-S2. For the complete graphs such as C_4 and C_{20} in Table 1, $\hat{x}_k/q^{m_k}$ is an unbiased estimator of x_k, while for the incomplete graphs:*

$$\widetilde{x}_k = \frac{\hat{x}_k + \sum_{C_i : C_k \subset C_i \& n_k = n_i} n_{ik}\hat{x}_i}{q^{m_k}} - \sum_{\substack{C_i : C_k \subset C_i \\ \& n_k = n_i}} n_{ik}\widetilde{x}_i \quad (2)$$

Proof 2 *When a pattern is a complete graph, the proof is the same as that of Proposition 1. When a pattern is an incomplete graph, $\hat{x}_k/q^{m_k}$ records the number of not only pattern C_k, but also the patterns that contain C_k as their subgraph and with the same node size,i.e.,$\{C_i : C_k \subset C_i \& n_k = n_i\}$. We name C_i as the parent pattern of C_k. In a sampled action diffusion graph, when enumerating an incomplete subgraph, it may be restored to any of its parent patterns. Thus to obtain $\widetilde{x}_k$, we need to first sum the approximate values of C_k and all its parents, and then divide by the sampling probability, q^{m_k}, to get an unbiased estimation (the first part of Eq. (2)), and finally subtract the unbiased value of all the parent patterns (the second part of Eq. (2)). Notation n_{ik} is the times that C_k is contained in C_i. For example, in Table 1, C_6 appears two times in C_{14}. The unbiased value, $\widetilde{x}_i$, of each parent pattern can be estimated in the same way iteratively, until the pattern itself is a complete graph.*

StructInf-S3. StructInf-S3 combines the above two approaches by not only sampling diffusion edges when building the action diffusion graph, but also sampling the action nodes when enumerating pattern instances.

Experiment

The dataset and code are available online now.[3]

Experimental Setup. We perform the evaluation on a dataset collected from Sina Weibo[1], The network consists of 1,776,950 users as nodes and 308,489,739 "following" relationships as edges, with the maximal degree d_{max} as 2,875. We use 23,755,810 tweet/retweet actions to build action diffusion graphs of 3,472,004 diffusion edges. The action type set A contains 300,000 types (i.e., the original tweets). Please refer to (Zhang et al. 2013) for details.

In the Weibo dataset, we first use the sampling algorithms to estimate the structural influence for the patterns in Table 1. We vary sampling probabilities and compare the error and time trade-off curves. The results of StructInf-Basic are used as ground truth. Please note that no existing methods can be

[3]http://aminer.org/structinf

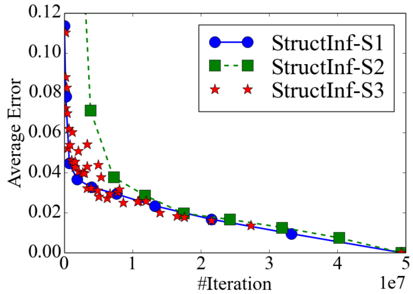


Figure 3: The trade-off between error and time by varying p and q. #Iteration is the times that Algorithm 2 is invoked.

directly used to estimate structural influence. Then we apply the extracted influence patterns to help retweet behavior prediction, to demonstrate the effectiveness of influence structures.

Experimental Results. We use relative error,

$$U_{\mathrm{IP}_k} = \frac{|\widetilde{\mathrm{IP}}_k - \mathrm{IP}_k|}{\mathrm{IP}_k},$$

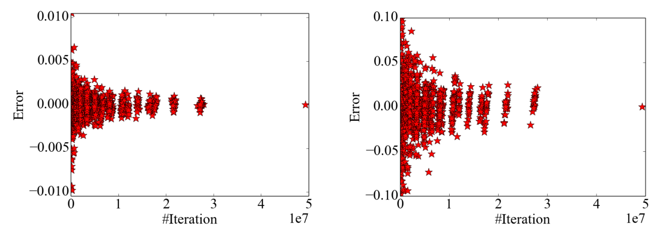
where IP_k is the exact/actual estimation and $\widetilde{\mathrm{IP}}_k$ is the approximate estimation, to measure how far the approximate estimation is from the exact estimation (Ahmed et al. 2014).

We present the optimal estimation of structural influence, IP_k, with k from 1 to 20 in Table 1. The results are obtained by executing StructInf-S3 with $\tau = 25$ hours, $q = 0.9$, $p_x = 0.6$ and $p_y = 0.1$, where p_x and p_y are the node sampling probabilities for estimating x_k and y_k respectively, and q is for sampling edges, and is the same for estimating x_k and y_k. Because the ratio between active and inactive instances is about 1:700, x_k can be estimated much faster than y_k. Thus we can first try several parameters to find the optimal values of q and p_x, and then estimate p_y approximately by $p_x \times \sqrt[n_k]{\frac{\widetilde{x}_k}{\widetilde{y}_k}}$, that is derived from the variance. We estimate each $\widetilde{x}_k$ and $\widetilde{y}_k$ by averaging the results of 10 independent runs, and based on which to calculate $\widetilde{\mathrm{IP}}_k$. Table 1 shows that each $\widetilde{\mathrm{IP}}_k$ is very close to the exact value. Most of the relative errors are around 1.0% and the worst case is about 5.0%. We also find that the top influential patterns are those with more nodes or edges than others (the numbers that are in boldface). To get the results in Table 1, the exact method (StructInf-Basic) requires 19 hours, while the sampling method (StructInf-S3) requires only 1.8 hours.

Trade-off between Error and Time. To compare the performance of different sampling methods, we show how accuracy and speed vary by varying the sampling probabilities. Specifically, we first tune the sampling probabilities. We try p in the range $\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$ for StructInf-S1, try q in the same range for StructInf-S2, and try all the possible combinations of p and q in the same range for StructInf-S3. Second, for each configuration of p and q, we calculate the average error of $\widetilde{x}_k$ over all the influence patterns, and further average them over 10 independent sampling results. Third, we use the number of iterations that Algorithm 2 is invoked as the metric to measure how fast an

Table 1: All structural influence patterns formed by 2, 3, and 4 nodes and the structural influence estimation results (%). The red nodes represent active friends who performed an action before and the white nodes represent the target user.

k	C_k	IP_k	$\widetilde{\mathrm{IP}}_k$	U_{IP_k}	k	C_k	IP_k	$\widetilde{\mathrm{IP}}_k$	U_{IP_k}
1		0.066	0.066	0.020	11		0.038	0.038	0.720
2		0.074	0.074	0.085	12		0.186	0.186	0.088
3		0.111	0.110	0.425	13		0.399	0.392	1.785
4		0.307	0.304	0.928	14		0.063	0.062	0.616
5		0.069	0.069	0.530	15		0.619	**0.615**	0.548
6		0.091	0.090	0.358	16		0.444	**0.439**	1.378
7		0.067	0.067	0.236	17		0.070	0.070	0.074
8		0.106	0.099	5.852	18		0.420	**0.416**	0.890
9		0.381	0.388	1.666	19		0.662	**0.645**	2.696
10		0.165	0.162	1.128	20		0.485	**0.479**	1.239



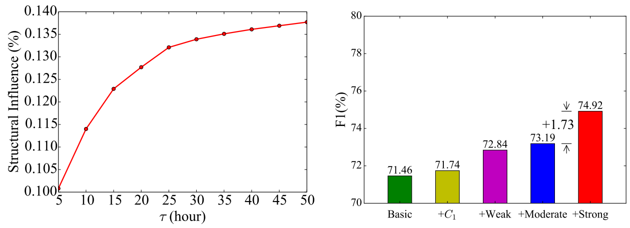
(a) C_1 (b) C_4

Figure 4: Convergence of the relative errors of patterns C_1 and C_4 by StructInf-S3.

approximate estimation will be. The more iterations we run, the more influence patterns will be sampled.

Figure 3 plots all the (relative-error, #iterations) pairs when varying p and q. For each method, the sampling parameters change from 0.1 to 1.0 incrementally from the upper-left to the lower-right points. Because StructInf-S3 has two tunable parameters, the points cannot be connected by one single line. From the results, we can see that first, the error curves of al the methods almost follow exponential distributions. The average error drops dramatically when p increases at the very beginning, and then changes slowly when p gets larger than 0.5. Second, StructInf-S1 performs better than StructInf-S2 because its curve is below that of StructInf-S2, implying that it needs less iterations to achieve the same error. Finally, the resultant points of StructInf-S3 are more concentrated in the bottom-left corner than those of StructInf-S1 and StructInf-S2, suggesting that StructInf-S3 is less sensitive to the parameters. This probably because StructInf-S3 combines the power of StructInf-S1 and StructInf-S2, thus achieves a more stable performance.

Unbiasedness. We take StructInf-S3 as an example to study the properties of the sampling distribution. Specifically, for each pair of q and p, we run 20 independent samplings and plot the relative error of each run. From Figure 4, we can see that the sampling distributions of different



(a) τ (b) Retweet prediction

Figure 5: (1) Effect of τ on structural influence; (2) Performance of retweet behavior prediction.

patterns are all centered over the line with relative error as 0, which implies the unbiased property of the estimations. In addition, influence patterns with fewer nodes and edges can achieve better performance with few iteration times (i.e., samples). We see that the simplest pattern C_1 obtains 0.5% relative error with few samples, while C_4 needs more samples to achieve the same error rate, because the sampling probability $p(c_i) = p^m q^n$ decreases with m and n, making the variance larger than that of the simple patterns.

Effect of Time Delay τ. We study how time delay τ affects structural influence. Figure 5(a) shows the changes of structural influence by varying τ. The Y-axis is the structural influence of C_1, which is the basic component of all the other patterns. We can see that social influence increases quickly over time and gradually becomes stable after τ gets larger than 25 hours, implying that social influence decays over time. Thus, we set τ to 25 hours on the Weibo dataset.

Application Improvements. We demonstrate how the discovered influence patterns can help improve the application of retweet prediction. The goal is, given an action triple (v, a, t), predict whether user v will retweet the tweet a at time t. Basically, for each observed action in the dataset, we treat it as a positive instance. For each positive instance (v, a, t), if a follower u of user v did not retweet a before $t + \tau$, we treat $(u, a, t + \tau)$ as a negative instance. We uniformly sample a balanced dataset with equal number of positive and negative instances and train a binary classifier with logistic regression. We feed some basic features, such as the number of followees/followers, gender, verification status of the user, to the classifier. We aim at investigating whether the structural influence patterns can improve prediction performance based on the basic features. Additionally, we add structural influence patterns as features (the number of each pattern that is counted from the target action (v, a, t)). In particular, we divide the patterns into four groups: C_1(i.e., the number of active neighbors), *weak* ($\widetilde{\mathrm{IP}}_k < 0.1$), *moderate* ($0.1 \leq \widetilde{\mathrm{IP}}_k < 0.3$), and *strong* ($\widetilde{\mathrm{IP}}_k > 0.3$). We respectively add the basic features and pattern groups one by one and evaluate the increase of the predictive performance. A larger increase means a higher predictive power. From the results in Figure 5(b), we observe that weak patterns can improve a lot upon basic features and the number of active neighbors (+1.1% in terms of F1), indicating the effect of structural influence on retweet behaviors. Furthermore, significant increase on F1 score is observed when adding strong patterns upon moderate patterns (+1.73%), while no evident increase is observed when adding moderate patterns, implying that the discovered significant structural influence patterns can benefit a lot on predicting retweet behaviors.

Related Work

Structural Influence. A few research examined the structural characteristics of social influence. Ugander et al. (2012) firstly studied structural diversity and found that the possibility that a user joins Facebook is positively affected by the diverse structure of the friends who have joined Facebook. Lately quite a few work has been conducted based on this idea in various scenarios (Fang et al. 2014; Kloumann et al. 2015; Qiu et al. 2016; Zhang et al. 2013). However, all the studies about structural diversity do not distinguish the particular influence structures, and our paper proposes an algorithm to enumerate all influence structures.
Influence Learning. The aim of influence learning is to associate each edge e_{uv} with a probability p_{uv} to represent the strength of influence exerted by u on v. (Kimura et al. 2011) were the first to learn influence by maximizing the likelihood of generating historical behavior data. Tang et al. (2009) and Liu et al. (2012) extended the influence probabilities to the topic level. Kutzkov et al. (Kutzkov et al. 2013) approximately estimated the pairwise influence in a behavior stream. In addition to learning pairwise influence, group influence is also studied by (Tang, Wu, and Sun 2013; Zhang et al. 2014). To the best of our knowledge, this is the first attempt to define and estimate structural influence.
Subgraph Mining/Sampling. Traditional research counted triangles (Jha, Seshadhri, and Pinar 2013; Pavan et al. 2013), 4-node subgraphs (Jha, Seshadhri, and Pinar 2015), any subgraphs (Ahmed et al. 2014; Wernicke 2006), or specified ones such as cliques, stars, chains, and cascading patterns (Koutra et al. 2015; Leskovec, Singh, and Kleinberg 2006). However, the methods cannot be directly applied to our problem, because the behavior data is streaming and the structural influence is beyond the frequency of a subgraph.

Conclusion

We present the concept of structural influence and propose a sampling algorithm to quickly estimate the influence probabilities of different structures from social stream, with theoretical proof of unbiasedness properties. Experiments on a large Weibo data show that, compared to the exact solution, the third sampling method can achieve the best performance, with a $10\times$ speedup and an average error rate of only 1.0%. We also demonstrate the effectiveness of the extracted high influential patterns on retweet behavior prediction.

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References

Ahmed, N. K.; Duffield, N.; Neville, J.; and Kompella, R. 2014. Graph sample and hold: A framework for big-graph analytics. In *KDD'14*, 1446–1455.

Anagnostopoulos, A.; Kumar, R.; and Mahdian, M. 2008. Influence and correlation in social networks. In *KDD'08*, 7–15.

Bond, R. M.; Fariss, C. J.; Jones, J. J.; Kramer, A. D. I.; Marlow, C.; Settle, J. E.; and Fowler, J. H. 2012. A 61-million-person experiment in social influence and political mobilization. *Nature* 489:295–298.

Fang, Z.; Zhou, X.; Tang, J.; Shao, W.; Fong, A.; Sun, L.; Ding, Y.; Zhou, L.; and Luo, J. 2014. Modeling paying behavior in game social networks. In *CIKM'14*, 411–420.

Fowler, J. H.; Christakis, N. A.; et al. 2008. Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the framingham heart study. *BMJ* 337:a2338.

Goyal, A.; Bonchi, F.; and Lakshmanan, L. V. 2010. Learning influence probabilities in social networks. In *WSDM'10*, 241–250.

Horvitz, D. G., and Thompson, D. J. 1952. A generalization of sampling without replacement from a finite universe. *Journal of the American Statistical Association* 47(260):663–685.

Jha, M.; Seshadhri, C.; and Pinar, A. 2013. A space efficient streaming algorithm for triangle counting using the birthday paradox. In *KDD'13*, 589–597.

Jha, M.; Seshadhri, C.; and Pinar, A. 2015. Path sampling: A fast and provable method for estimating 4-vertex subgraph counts. In *WWW'15*, 495–505.

Kashtan, N.; Itzkovitz, S.; Milo, R.; and Alon, U. 2004. Efficient sampling algorithm for estimating subgraph concentrations and detecting network motifs. *Bioinformatics* 20(11):1746–1758.

Kempe, D.; Kleinberg, J.; and Tardos, E. 2003. Maximizing the spread of influence through a social network. In *KDD'03*, 137–146.

Kimura, M.; Saito, K.; Ohara, K.; and Motoda, H. 2011. Learning information diffusion model in a social network for predicting influence of nodes. *Intell. Data Anal.* 15:633–652.

Kloumann, I.; Adamic, L.; Kleinberg, J.; and Wu, S. 2015. The lifecycles of apps in a social ecosystem. In *WWW'15*, 581–591.

Koutra, D.; Kang, U.; Vreeken, J.; and Faloutsos, C. 2015. Summarizing and understanding large graphs. *Statistical Analysis and Data Mining* 8(3):183–202.

Kutzkov, K.; Bifet, A.; Bonchi, F.; and Gionis, A. 2013. Strip: stream learning of influence probabilities. In *KDD'13*, 275–283.

Leskovec, J.; Singh, A.; and Kleinberg, J. 2006. Patterns of influence in a recommendation network. In *PAKDD'06*, 380–389.

Liu, L.; Tang, J.; Han, J.; and Yang, S. 2012. Learning influence from heterogeneous social networks. *DMKD* 25(3):511–544.

Pavan, A.; Tangwongsan, K.; Tirthapura, S.; and Wu, K.-L. 2013. Counting and sampling triangles from a graph stream. *VLDB* 6(14):1870–1881.

Qiu, J.; Li, Y.; Tang, J.; Lu, Z.; Ye, H.; Chen, B.; Yang, Q.; and Hopcroft., J. 2016. The lifecycle and cascade of wechat social messaging groups. In *WWW'16*.

Shi, X.; Leskovec, J.; and McFarland, D. A. 2010. Citing for high impact. In *JCDL'10*, 49–58.

Tang, J.; Sun, J.; Wang, C.; and Yang, Z. 2009. Social influence analysis in large-scale networks. In *KDD'09*, 807–816.

Tang, J.; Wu, S.; and Sun, J. 2013. Confluence: Conformity influence in large social networks. In *KDD'13*, 347–355.

Ugander, J.; Backstrom, L.; Marlow, C.; and Kleinberg, J. 2012. Structural diversity in social contagion. *PNAS* 109(16):5962–5966.

Wernicke, S. 2006. Efficient detection of network motifs. *TCBB* 3(4):347–359.

Yan, X., and Han, J. 2002. gspan: Graph-based substructure pattern mining. In *ICDM'02*, 721–724.

Zhang, J.; Liu, B.; Tang, J.; Chen, T.; and Li, J. 2013. Social influence locality for modeling retweeting behaviors. In *IJCAI'13*, 2761–2767.

Zhang, J.; Tang, J.; Zhuang, H.; Leung, C. W.-K.; and Li, J. 2014. Role-aware conformity influence modeling and analysis in social networks. In *AAAI'14*, 1–7.

Zhang, J.; Fang, Z.; Chen, W.; and Tang, J. 2015. Diffusion of following links in microblogging networks. *TKDE* 27(8):2093–2106.