Influence Maximization in Dynamic Social Networks

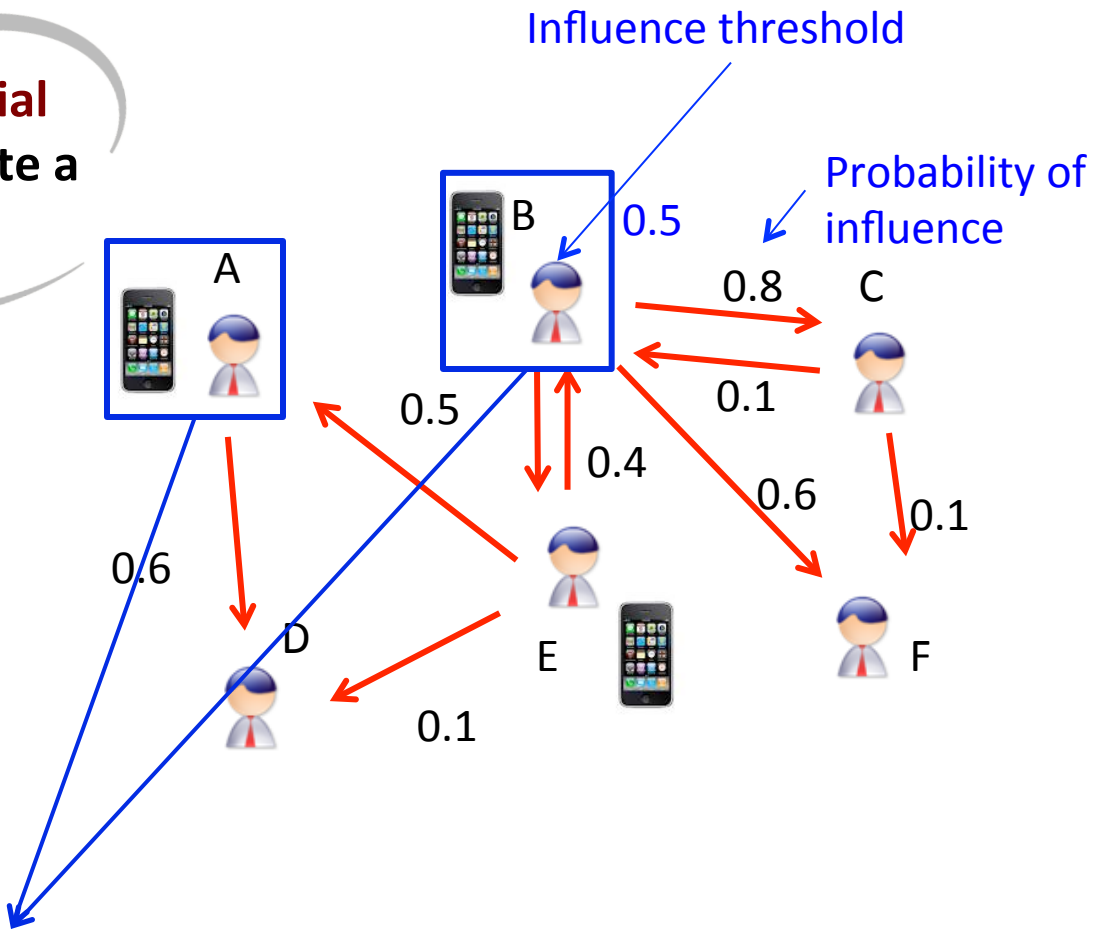
Honglei Zhuang, Yihan Sun, Jie Tang,
Jialin Zhang, Xiaoming Sun

中国科学院计算技术研究所
INSTITUTE OF COMPUTING TECHNOLOGY, CHINESE ACADEMY OF SCIENCES

Influence Maximization

How to find influential users to help promote a new product?

Marketer Alice



Find *K* nodes (users) in a social network that could maximize the spread of influence (Domingos, 01; Richardson, 02; Kempe, 03)

Influence Maximization

- Problem[1]
  - Initially all users are considered inactive
  - Then the chosen users are activated, who may further influence their friends to be active as well
- Models
  - Linear Threshold model
  - Independent Cascading model

[1] D. Kempe, J. Kleinberg, and E. Tardos. Maximizing the spread of influence through a social network. KDD'03, pages 137–146, 2003.

Approximate Solution

- NP-hard [1]
  - Linear Threshold Model
  - Independent Cascading Model

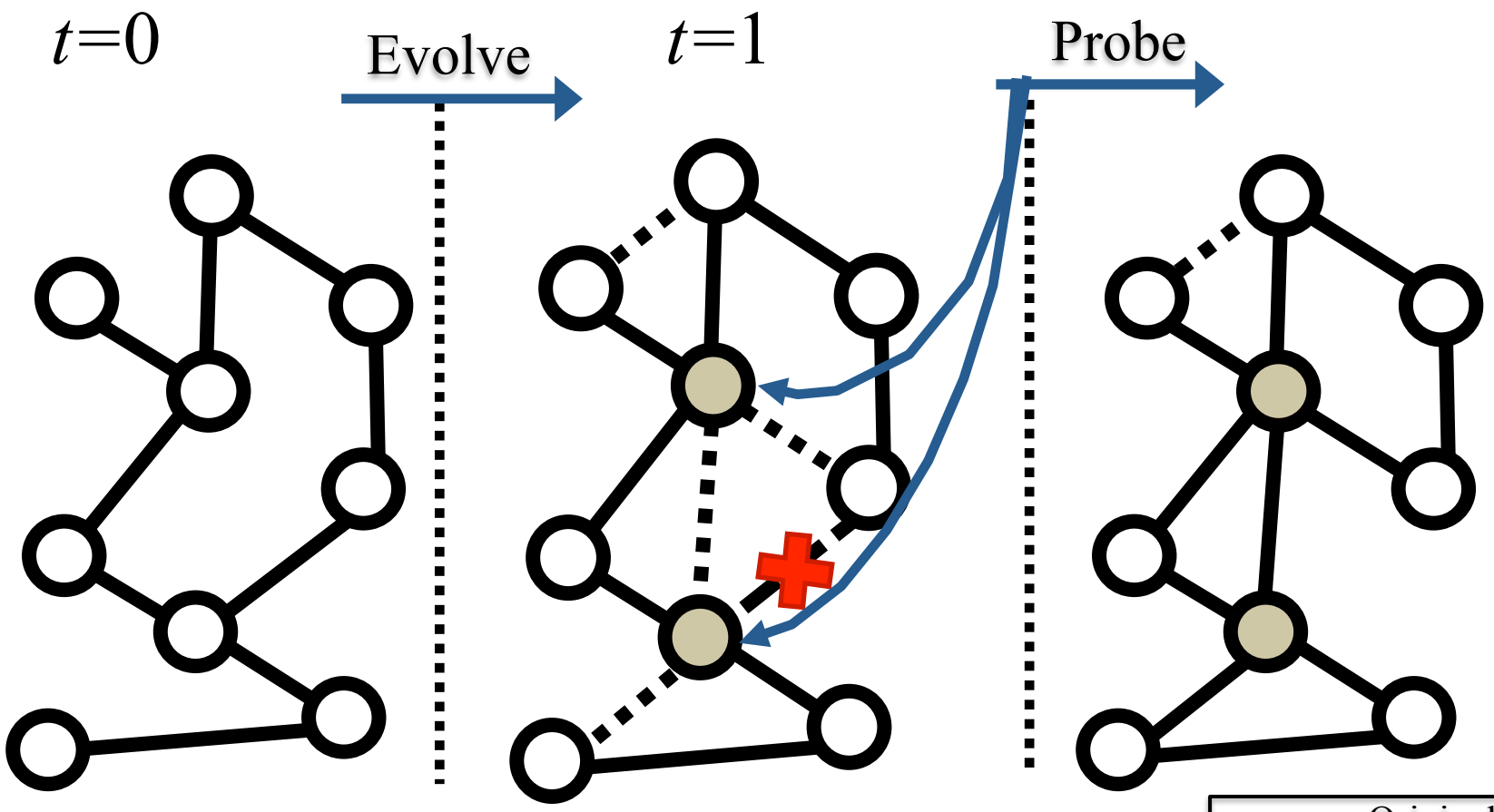
The problem is solved by optimizing a monotonic submodular function

$$f(S \cup \{v\}) - f(S) \geq f(T \cup \{v\}) - f(T)$$

- Kempe Prove that approximation algorithms can guarantee that the influence spread is within(1-1/e) of the optimal influence spread.
  - Verify that the two models can outperform the traditional heuristics
- Recent research focuses on the efficiency improvement
  - [2] accelerate the influence procedure by up to 700 times
- It is still challenging to extend these methods to large data sets

[1] D. Kempe, J. Kleinberg, and E. Tardos. Maximizing the spread of influence through a social network. KDD'03, pages 137–146, 2003.
[2] J. Leskovec, A. Krause, C. Guestrin, C. Faloutsos, J. VanBriesen, and N. Glance. Cost-effective outbreak detection in networks. KDD'07, pages 420–429, 2007.

Influence Maximization in Dynamic Networks



About **6 million** links changed on Weibo network

Weibo API limitation: $\leqslant$ **450** times/hr

Problem

- Input: For a dynamic social network $\{G^0, \ldots, G^t\}$, we have observed G^0, but for all $t>0$, G^t is unknown

- Problem: To probe b nodes, observe their neighbors to obtain an observed network $\hat{G}^t$ from $\hat{G}^{t-1}$ / G^0, such that influence maximization on the real network G^t can be approximated by that on the observed network.

- Challenge: How to find the k influential users, if we only partially observe the update of the social network?

Basic Idea

- Estimate how likely the neighborhood of a node will change in a dynamic social network
  - Probe nodes that change a lot
- Estimate how much the influence spread can be improved by probing a node
  - Probe the one maximizes the improvement

Methodologies and Results

Preliminary Theoretical Analysis

- Formal definition of loss

Max seed set on fully observed network

$$\mathrm{l} = E_{G|\hat{G}}\left[\left|Q(S^*) - Q(T^*)\right|\right]$$

Max seed set on partially observed network

- With an specified evolving graph model
  - At each time stamp an edge is chosen uniformly
  - and its head will point to a node randomly chosen with probability proportional to the in-degree

Preliminary Theoretical Analysis

- Error bound of Random probing strategy

$$\begin{aligned} \ell^t_{Rand} &\leq \sum_{x \in S^*} \frac{4np}{m} \left[\hat{d}^{t'_x}(x) + \frac{1}{4} p \left(\hat{d}^{t'_x}(x) \right)^2 \right] \\ &+ \sum_{x \in T^*} \frac{4np}{m} \left[\hat{d}^{t'_x}(x) + \frac{1}{4} p \left(\hat{d}^{t'_x}(x) \right)^2 \right] \end{aligned}$$

- Error bound of Degree weighted probing strategy

$$\ell^t_{DegRR} \leq 16pk + 2p^2 \left[\sum_{x \in S^*} \hat{d}^{t'_x}(x) + \sum_{x \in T^*} \hat{d}^{t'_x}(x) \right]$$

- In most cases, degree weighted probing strategy performs better than random probing strategy

Maximum Gap Probing

- Basic Idea
  - Estimate how much the influence spread can be improved by probing a node
  - Probe the one which maximizes the improvement
- Formally,
  - For a given tolerance probability ε
  - The minimum value β that satisfies the following inequality is defined as performance gap $\beta(v)$

$$P\left[\hat{Q}_v\left(S'_o(v)\right) - \hat{Q}_v\left(S_o\right) \geq \beta\right] \leq \varepsilon$$

Best solution if v is probed — **Best solution before probing**

*To simplify problem, define the quality function as the sum of degree in the seed set.

Maximum Gap Probing

- Assume the degree of a node is a martingale. We can estimate the degree gap of each node by

$$P\left[d^t(v) - d^{t-c_v}(v) \geq \sqrt{-2c_v \ln \varepsilon} \right] \leq \varepsilon$$

Last time when v is probed **Defined as z_v**

- Considering the node to probe is in/not in the current seed set.

$$\beta(v) = \begin{cases} \max\left\{0, \hat{d}(v) + z_v - \min_{w \in S_o} \hat{d}(w)\right\}, v \notin S_O \\ \max\left\{0, \max_{u \notin S_o} \hat{d}(u) - \hat{d}(v) + z_v\right\}, v \in S_O \end{cases}$$

- Each time, choose the one with maximum gap $\beta(v)$ to probe

MaxG Algorithm

Input: G^0, T, ϵ, b
Output: Seed set S^t at $t = 1, 2, \cdots, T$

$\hat{G} \leftarrow G^0$; $\forall v \in V, c_v \leftarrow 0$;
for $t = 1$ **to** T **do**
  $\forall v \in V, c_v \leftarrow c_v + 1$;
  for b *times* **do**
    $S_o \leftarrow k$ nodes with maximum $\hat{d}_{in}(v)$;
    $\hat{d}_{max} = \max_{u \notin S_o} \hat{d}_{in}(u)$;
    $\hat{d}_{min} = \min_{w \in S_o} \hat{d}_{in}(w)$;
    foreach $v \in V$ **do**
      $z_v \leftarrow \sqrt{-2c_v \ln \epsilon}$;
      if $v \in S$ **then**
      $\beta_v \leftarrow \max\left\{0, \hat{d}_{max} - \hat{d}_{in}(v) + z_v\right\}$;
      else $\beta_v \leftarrow \max\left\{0, \hat{d}_{in}(v) + z_v - \hat{d}_{min}\right\}$;
    $v^* \leftarrow \arg\max_{v \in V} \beta_v$, $c_{v^*} \leftarrow 0$;
    Probe v^* in G^t and update $\hat{G}$;
  // Degree discount heuristics
  $S^t \leftarrow \emptyset$;
  for k *times* **do**
    $v^* \leftarrow \arg\max_{v \in V \setminus S^t} \hat{h}_{S^t}(v)$;
    $S^t \leftarrow S^t \cup \{v^*\}$;
    foreach *neighbor* u *of* v^* **do**
      Update $\hat{h}_{S^t}(u)$;
  Output S^t;

Finding nodes to probe by maximizing the degree gap

Perform the standard greedy algorithm (degree discount heuristics) for influence maximization

Experiment Setup

- Data sets

Data sets	#Users	#Relationships	#Time stamps
Synthetic	500	12,475	200
Twitter	18,089,810	21,097,569	10
Coauthor[1]	1,629,217	2,623,832	27

- Evaluation
  - Take optimal seed set S' obtained from partially observed network
  - Calculate its influence spread on real network

[1] http://arnetminer.org/citation

Experiment Setup

- Comparing methods
  - *Rand, Enum:* Uniform probing
  - *Deg, DegRR:* Degree-weighted probing
  - *BEST:* Suppose network dynamics fully observed
- Configurations
  - Probing budget:
    - b=1,5 for Synthetic; b=100,500 for Twitter and Coauthor
  - Seed set size for influence maximization:
    - k=30 for Synthetic; k=100 for Twitter and Coauthor
  - Independent Cascade Model, with uniform p=0.01

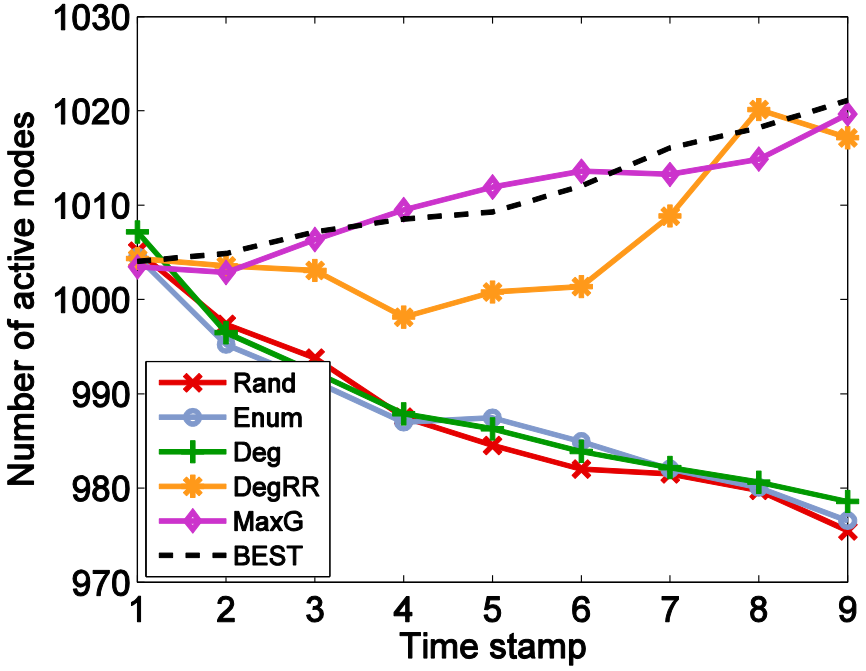
Experimental Results

- Average influence spread

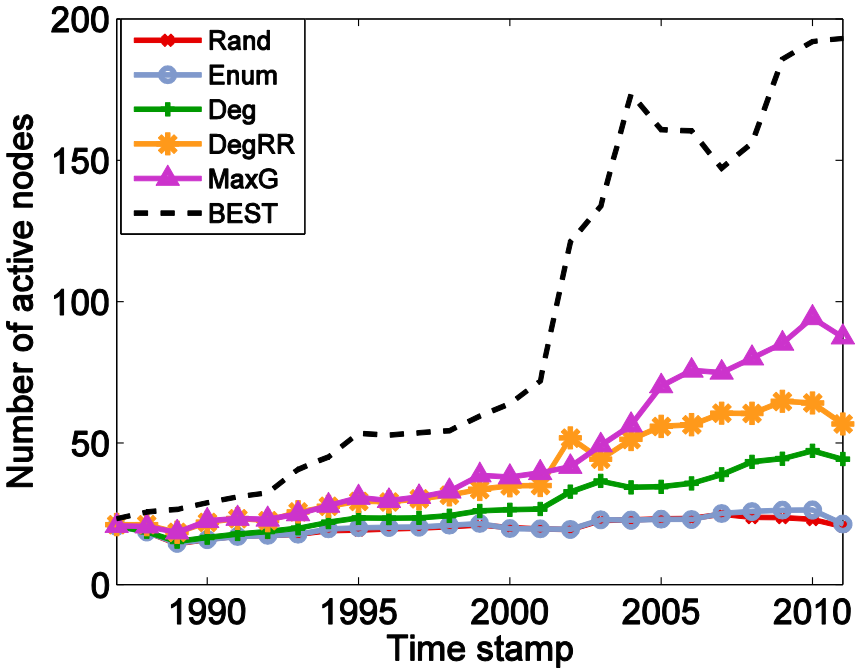
Data Set	b	Rand	Enum	Deg	DegRR	MaxG	BEST
Synthetic	1	13.83	13.55	13.78	14.30	**14.79**	15.95
	5	15.07	15.33	15.09	15.40	**15.60**	
Twitter	100	987.74	987.62	988.41	1001.47	**1005.12**	1011.15
	500	987.45	987.67	988.36	1006.38	**1010.61**	
Coauthor	100	20.34	20.82	28.67	38.94	**45.51**	91.51
	500	20.35	22.93	44.27	56.68	**61.74**	

The large, the best

Influence Maximization Results (b=100)

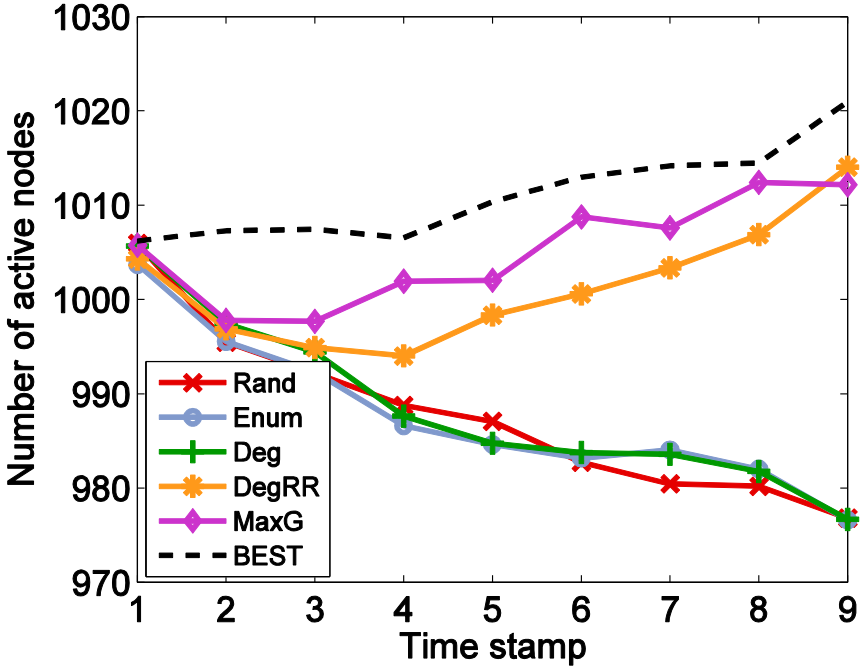


Twitter

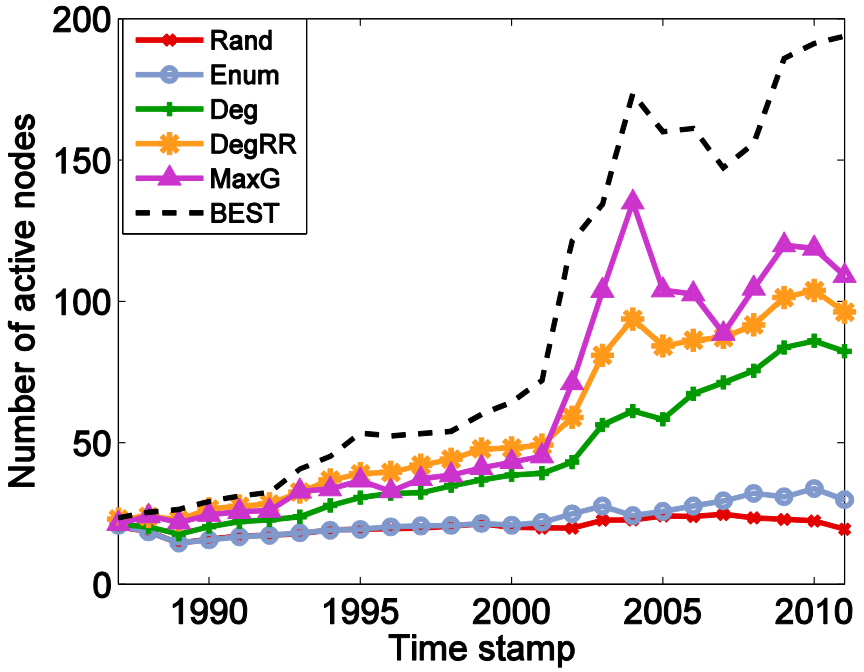


Coauthor

Influence Maximization Results (b=500)



Twitter



Coauthor

Conclusions

Conclusions

- Propose a probing algorithm to partially update a dynamic social network, so as to guarantee the performance of influence maximization in dynamic social networks

- Future work include:
  - Online updating seed set in dynamic social networks
  - Probing for other applications, e.g. PageRank[1]

[1] B. Bahmani, R. Kumar, M. Mahdian, and E. Upfal. PageRank on an evolving graph. In KDD, pages 24–32, 2012.

Thank you!